

# WIMP dark matter as radiative neutrino mass messenger

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**ABSTRACT:** The minimal seesaw extension of the Standard  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  Model requires two electroweak singlet fermions in order to accommodate the neutrino oscillation parameters at tree level. Here we consider a next to minimal extension where light neutrino masses are generated radiatively by two electroweak fermions: one singlet and one triplet under  $SU(2)_L$ . These should be odd under a parity symmetry and their mixing gives rise to a stable weakly interactive massive particle (WIMP) dark matter candidate. For mass in the GeV–TeV range, it reproduces the correct relic density, and provides an observable signal in nuclear recoil direct detection experiments. The fermion triplet component of the dark matter has gauge interactions, making it also detectable at present and near future collider experiments.

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## 1 Introduction

Despite the successful discovery of the Higgs boson, so far the Large Hadron Collider (LHC) has not discovered any new physics, so neutrino physics remains, together with dark matter, as the main motivation to go beyond the Standard Model (SM). Neutrino oscillation experiments indicate two different neutrino mass squared differences [1, 2]. As a result at least two of the three active neutrino must be massive, though the oscillation interpretation is compatible with one of the neutrinos being massless. In the Standard Model neutrinos have no mass at the renormalizable level. However they can get a Majorana mass by means of the dimension-5 Weinberg operator,

$$\frac{c}{\Lambda} LH LH, \quad (1.1)$$

where  $\Lambda$  is an effective scale,  $c$  a dimensionless coefficient and  $L$  and  $H$  denote the lepton and Higgs isodoublets, respectively. This operator should be understood as encoding new physics associated to heavy “messenger” states whose fundamental renormalizable interactions should be prescribed. The smallness of neutrino masses compared to the other fermion masses, suggests that the messenger scale  $\Lambda$  must be much higher than the electroweak scale if the coefficient  $c$  in equation 1.1 is of  $\mathcal{O}(1)$ . For example, the scale  $\Lambda$  should be close to the Grand Unification scale if  $c$  is generated at tree level. One popular mechanism to generate the dimension-5 operator is the so-called *seesaw mechanism*. Its most general  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  realization is the so called “1-2-3” seesaw scheme [3] with singlet, doublet and triplet scalar  $SU(2)_L$  fields with vevs respectively  $v_1$ ,  $v_2$  and  $v_3$ . Assuming

$m$  extra singlet fermions (right-handed neutrinos), the “1-2-3” scheme is described by the  $(3+m) \times (3+m)$  matrix

$$M^\nu = \begin{pmatrix} Y_3 v_3 & Y_2 v_2 \\ Y_2^T v_2 & Y_1 v_1 \end{pmatrix}. \quad (1.2)$$

The vevs obey the seesaw relation

$$v_3 v_1 \sim v_2^2 \quad \text{with} \quad v_1 \gg v_2 \gg v_3, \quad (1.3)$$

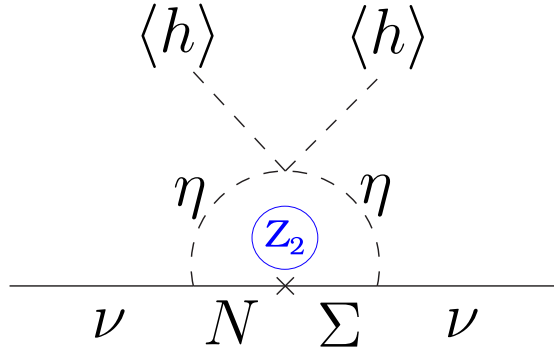
giving two contributions to the light neutrino masses  $Y_3 v_3 + v_2^2/v_1 Y_2 Y_1^{-1} Y_2^T$ , called respectively type-II and type-I seesaw. Assuming  $Y_3 = 0$ , namely no Higgs triplet <sup>1</sup>, the light neutrino masses arise only from the type-I seesaw contribution. In this case it is well known that in order to accommodate the neutrino oscillation parameters, at least two right-handed neutrinos are required, namely  $m \geq 2$ . We call the case  $m = 2$  minimal. Note that in this case one neutrino mass is zero and so the absolute neutrino mass scale is fixed. Typically the next to minimal case is to assume three sequential right-handed neutrinos, that is  $m = 3$ . An alternative seesaw mechanism is the so called type-III in which the heavy “right-handed” neutrino “messenger” states are replaced by  $SU(2)_L$  triplet fermions [4]. As for the type-I seesaw case, one must assume at least two fermion triplets (if only fermion triplets are present) in order to accommodate current neutrino oscillation data.

There is an interesting way to induce the dimension-5 operator by mimicking the seesaw mechanism at the radiative level. This requires the fermion messengers to be odd under an ad-hoc symmetry  $Z_2$  in order to accommodate a stable dark matter (DM) candidate. In this case one can have “scotogenic” [5] neutrino masses, induced by dark matter exchange. This trick can be realized either in type-I or type-III seesaw schemes [5, 6]. To induce Yukawa couplings between the extra fermions and the Standard Model leptons, one must include additional scalar doublets, odd under the assumed  $Z_2$  symmetry, and without vacuum expectation value. In order to complete the saga in this paper we propose a hybrid scotogenic construction which consists in having just one singlet fermion ( $m = 1$ ) but adding one triplet fermion as well. This also gives rise to light neutrino masses, calculable at the one loop level, as illustrated in figure 1 <sup>2</sup>. However, due to triplet–singlet mixing, the lightest combination of the neutral component of the fermion triplet and the singlet will be stable and can play the role of WIMP dark matter. We show that it provides a phenomenologically interesting alternative to all previous “scotogenic” proposals since here the dark matter can have sizeable gauge interactions. As a result, in addition to direct and indirect detection signatures, it can also be kinematically accessible to searches at present colliders such as the LHC.

Existing collider searches at LEP [7, 8] and LHC [9], set a nominal lower bound of  $\sim 100$  GeV for the masses of new charged particles. However, coannihilations present in the

<sup>1</sup>Note that in pure type-II seesaw, only one extra scalar field is required, in contrast with type-I, where at least two fermion singlets must be assumed.

<sup>2</sup>Note the scalar contributions come from the scalar and pseudoscalar pieces of the field  $\eta$ .



**Figure 1.** One loop realization for the Weinberg operator.

early universe, between the neutral and charged components, set the dark matter mass to be of the order of [6]

$$M_{\text{DM}} \simeq 2.7 \text{ TeV} \quad (1.4)$$

in order to explain the observed abundance [10]:

$$\Omega_{\text{DM}} h^2 = 0.1196 \pm 0.0031. \quad (1.5)$$

Radiative neutrino masses generated by at least two generations of fermion singlets or triplets have been studied in Ref. [11]. Here we focus on the radiative neutrino mass generation with one singlet and one triplet fermion which has interesting phenomenological consequences compared to the cases aforementioned cases. In our scenario, the dark matter candidate can indeed be observed not only in indirect but can also be kinematically accessible to current collider searches, and need not obey Eq. (1.4). Moreover, we will show that, in contrast to the proposed schemes in Refs. [5, 6] in our framework amplitudes leading naturally to direct detection processes appear at the tree level, thanks to singlet-triplet mixing effects.

The rest of this paper is organized as follows: in section 2 we introduce the new fields and interactions present in the model, making emphasis upon the mixing matrices and the radiative neutrino mass generation mechanism. Section 3 is devoted to numerical results on the phenomenology of dark matter in this model. An interesting feature of the model is the wide range of possible dark matter masses, ranging from 1 GeV to a few TeV. We also briefly discuss some the implications for LHC physics. In Section 4 we give our conclusions.

## 2 The model

Our model combines the ingredients employed in the models proposed in [5, 6] in such a way that it has a richer phenomenology than either [5] or [6].

### 2.1 The Model and the Particle Content

The new fields with respect to the Standard Model include one Majorana fermion triplet  $\Sigma$  and a Majorana fermion singlet  $N$  both with zero hypercharge and both odd under an ad-hoc symmetry  $Z_2$ . We also include a scalar doublet  $\eta$  with same quantum numbers as the

	Standard Model			Fermions		Scalars	
	$L$	$e$	$\phi$	$\Sigma$	$N$	$\eta$	$\Omega$
$SU(2)_L$	2	1	2	3	1	2	3
$Y$	-1	-2	1	0	0	1	0
$Z_2$	+	+	+	-	-	-	+

**Table 1.** Matter assignment of the model.

Higgs doublet, but odd under  $Z_2$ . In addition, we require that  $\eta$  not to acquire a vev. As a result, neutrino masses are not generated at tree level by a type-I/III seesaw mechanism. Instead they are one-loop calculable, from diagrams in Fig. 1. Furthermore, this symmetry forbids the decays of the lightest  $Z_2$  odd particle into Standard Model particles, which is a mixture of the neutral component of  $\Sigma$  and  $N$ . As a result this becomes a viable dark matter candidate. Note also that our proposed model does not modify quark dynamics, since neither of the new fields couples to quarks.

The fermion triplet, can be expanded as follows ( $\sigma_i$  are the Pauli matrices):

$$\Sigma = \Sigma_1 \sigma_1 + \Sigma_2 \sigma_2 + \Sigma_3 \sigma_3 = \begin{pmatrix} \Sigma_0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma_0 \end{pmatrix}, \quad (2.1)$$

where

$$\Sigma^+ = \frac{1}{\sqrt{2}} (\Sigma_1 + i\Sigma_2), \quad (2.2)$$

$$\Sigma^- = \frac{1}{\sqrt{2}} (\Sigma_1 - i\Sigma_2), \quad (2.3)$$

$$\Sigma^0 = \Sigma_3. \quad (2.4)$$

The  $Z_2$  is exactly conserved in the Lagrangian, moreover, it allows interactions between dark matter and leptons, in fact, this is the origin of radiative neutrino masses. The Yukawa couplings between the triplet and leptons play an important role in the dark matter production. Finally a triplet scalar  $\Omega$  is introduced in order to mix the neutral part of the fermion triplet  $\Sigma^0$  and the fermion singlet  $N$ . This triplet scalar field also has zero hypercharge and is even under the  $Z_2$  symmetry, thus, its neutral component can acquire a nonzero vev.

## 2.2 Yukawa Interactions and Fermion Masses

The most general  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  and Lorentz invariant Lagrangian is given as

$$\begin{aligned} \mathcal{L} \supset & -Y_{\alpha\beta} \bar{L}_\alpha e_\beta \phi - Y_{\Sigma\alpha} \bar{L}_\alpha C \Sigma^\dagger \tilde{\eta} - \frac{1}{4} M_\Sigma \text{Tr} [\bar{\Sigma}^c \Sigma] + \\ & -Y_\Omega \text{Tr} [\bar{\Sigma} \Omega] N - Y_{N\alpha} \bar{L}_\alpha \tilde{\eta} N - \frac{1}{2} M_N \bar{N}^c N + h.c., \end{aligned} \quad (2.5)$$

The  $C$  symbol stands for the Lorentz charge conjugation matrix  $i\sigma_2$  and  $\tilde{\eta} = i\sigma_2 \eta^*$ .

The Yukawa term  $Y_{\alpha\beta}$  is the SM Yukawa interaction for leptons, taken as diagonal matrix in the flavor basis<sup>3</sup>. On the other hand the Yukawa coupling  $Y_\Omega$  mixes the  $\Sigma$  and  $N$  fields and when the neutral part of the  $\Omega$  field acquire a vev  $v_\Omega$ , the dark matter particle can be identified to the lightest mass eigenstate of the mass matrix,

$$M_\chi = \begin{pmatrix} M_\Sigma & 2Y_\Omega v_\Omega \\ 2Y_\Omega v_\Omega & M_N \end{pmatrix}, \quad (2.6)$$

in the basis  $\psi^T = (\Sigma_0, N)$ . As a result one gets the following tree level fermion masses

$$m_{\chi^\pm} = M_\Sigma, \quad (2.7)$$

$$m_{\chi_1^0} = \frac{1}{2} \left( M_\Sigma + M_N - \sqrt{(M_\Sigma - M_N)^2 + 4(2Y_\Omega v_\Omega)^2} \right), \quad (2.8)$$

$$m_{\chi_2^0} = \frac{1}{2} \left( M_\Sigma + M_N + \sqrt{(M_\Sigma - M_N)^2 + 4(2Y_\Omega v_\Omega)^2} \right), \quad (2.9)$$

$$\tan(2\alpha) = \frac{4Y_\Omega v_\Omega}{M_\Sigma - M_N}, \quad (2.10)$$

where  $\alpha$  is the mixing angle between  $\Sigma_0$  and  $N$ . Here  $M_\Sigma$  and  $M_N$  characterize the Majorana mass terms for the triplet and the singlet, respectively. The  $M_\Sigma$  term is also the mass of the charged component of the  $\Sigma$  field, this issue is important because the mass splitting between  $\Sigma^\pm$  and the dark matter candidate will play a role in the calculation of its relic density. As we will see later, the splitting induced by  $v_\Omega$  allows us to relax the constraints on the dark matter coming from the existence of  $\Sigma^\pm$ .

### 2.3 Scalar potential and spectrum

The most general scalar potential, even under  $Z_2$ , including the fields  $\phi$ ,  $\eta$  and  $\Omega$  and allowing for spontaneous symmetry breaking, may be written as:

$$\begin{aligned} V_{\text{scal}} = & -m_1^2 \phi^\dagger \phi + m_2^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) \\ & + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\lambda_5}{2} (\phi^\dagger \eta)^2 + h.c. - \frac{M_\Omega^2}{4} \text{Tr} (\Omega^\dagger \Omega) + (\mu_1 \phi^\dagger \Omega \phi + h.c.) \\ & + \lambda_1^\Omega \phi^\dagger \phi \text{Tr} (\Omega^\dagger \Omega) + \lambda_2^\Omega (\text{Tr} (\Omega^\dagger \Omega))^2 + \lambda_3^\Omega \text{Tr} ((\Omega^\dagger \Omega)^2) + \lambda_4^\Omega (\phi^\dagger \Omega) (\Omega^\dagger \phi) \\ & + (\mu_2 \eta^\dagger \Omega \eta + h.c.) + \lambda_1^\eta \eta^\dagger \eta \text{Tr} (\Omega^\dagger \Omega) + \lambda_4^\eta (\eta^\dagger \Omega) (\Omega^\dagger \eta), \end{aligned} \quad (2.11)$$

where the fields  $\eta$ ,  $\phi$  and  $\Omega$ , can be written as follows:

$$\begin{aligned} \eta &= \begin{pmatrix} \eta^+ \\ (\eta^0 + i\eta^A)/\sqrt{2} \end{pmatrix}, \\ \phi &= \begin{pmatrix} \varphi^+ \\ (h_0 + v_h + i\varphi)/\sqrt{2} \end{pmatrix}, \\ \Omega &= \begin{pmatrix} (\Omega_0 + v_\Omega) & \sqrt{2}\Omega^+ \\ \sqrt{2}\Omega^- & -(\Omega_0 + v_\Omega) \end{pmatrix}, \end{aligned} \quad (2.12)$$

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<sup>3</sup>We can always go to this basis with a unitary transformation.

where  $v_h$  and  $v_\Omega$  are the vevs of  $\phi$  and  $\Omega$  fields respectively. We have three charged fields one of which is absorbed by the  $W$  boson, three CP-even physical neutral fields, and two CP-odd neutral fields one of which is absorbed by the  $Z$  boson <sup>4</sup>.

Let us first consider the charged scalar sector. The charged Goldstone boson is a linear combination of the  $\varphi^+$  and the  $\Omega^+$ , changing the definition for the  $W$  boson mass from that in the Standard Model :  $M_W = \frac{g}{2}\sqrt{v_h^2 + v_\Omega^2}$ . Note that this places a constraint on the vev of  $v_\Omega$  from electroweak precision tests [12, 13], one can expect roughly this vev to be less than 7 GeV, in order to keep the  $M_Z = \frac{\sqrt{g^2 + g'^2}}{2}v_h$  in the experimental range, and alter the  $M_W$  value inside the experimental error band.

Apart from the  $W$  boson, the two charged scalars have mass:

$$M_\pm^2 = 2\mu_1 (v_h^2 + v_\Omega^2) / v_\Omega, \quad (2.13)$$

$$m_{\eta^\pm}^2 = m_2^2 + \frac{1}{2}\lambda_3 v_h^2 + 2\mu_2 v_\Omega + (2\lambda_1^\eta + \lambda_4^\eta) v_\Omega^2. \quad (2.14)$$

Notice that the nonzero vacuum expectation value  $v_\Omega \neq 0$  will play an important role in generating the novel phenomenological effects of interest to us (see below). Now let us consider the neutral part: the minimization conditions of the Higgs potential allow vevs for the neutral part of the usual  $\phi$  field as well as for the neutral part of the  $\Omega$  field. The mass matrix for neutral scalar eigenstates in the basis  $\Phi^T = (h_0, \Omega_0)$  is:

$$\mathcal{M}_s^2 = \begin{pmatrix} \lambda_1 v_h^2 + \frac{t_h}{v_h} & -2\mu_1 v_h + 4v_h v_\Omega \left( \lambda_1^\Omega + \frac{\lambda_4^\Omega}{2} \right) \\ -2\mu_1 v_h + 4v_h v_\Omega \left( \lambda_1^\Omega + \frac{\lambda_4^\Omega}{2} \right) & \frac{\mu_1 v_h^2}{v_\Omega} + 16v_\Omega^2 (2\lambda_2^\Omega + \lambda_3^\Omega) + \frac{t_\Omega}{v_\Omega} \end{pmatrix}, \quad (2.15)$$

where  $t_h$  and  $t_\Omega$  are the tadpoles for  $h_0$  and  $\Omega_0$  and are described in Appendix A.2. The presence of the vev  $v_\Omega$  induces the mixing between  $h_0$  and  $\Omega_0$ . The corresponding eigenvalues give us the masses of the Standard Model Higgs doublet and the second neutral scalar both labelled as  $S_i^0$ .

On the other hand, the  $\eta$  field does not acquire vev, therefore, the mass eigenvalues of the neutral  $\eta^0$ , charged  $\eta^\pm$  and pseudoscalar  $\eta^A$  are decoupled. The spectrum for  $\eta^0$  and  $\eta^A$  fields is:

$$m_{\eta^0}^2 = m_{\eta^\pm}^2 + \frac{1}{2}(\lambda_4 + \lambda_5) v_h^2 - 4\mu_2 v_\Omega, \quad (2.16)$$

$$m_{\eta^A}^2 = m_{\eta^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5) v_h^2 - 4\mu_2 v_\Omega. \quad (2.17)$$

## 2.4 Radiative Neutrino Masses

In this model, neutrino masses are generated at one loop. The dark matter candidate particle acts as a messenger for the masses. The relevant interactions for radiative neutrino mass generation arise from from Eqs. (2.5) and (2.11) and can be written in terms of the tree

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<sup>4</sup>Remember that the neutral part of  $\Omega$  field is real, so it does not contribute to the CP-odd sector.

level mass eigenstates. Symbolically, one can rewrite the relevant terms for this purpose as:

$$\begin{aligned} L \Sigma \eta &\longrightarrow h_{ij} \nu_i \chi_j^0 \eta_0 \quad , \quad h_{ij} \nu_i \chi_j^0 \eta_A \\ L \eta N &\longrightarrow h_{ij} \nu_i \chi_j^0 \eta_0 \quad , \quad h_{ij} \nu_i \chi_j^0 \eta_A \\ (\phi^\dagger \eta)^2 &\longrightarrow [(h + v_h) \eta_0]^2 \quad , \quad [(h + v_h) \eta_A]^2 \end{aligned} \quad (2.18)$$

Here the field  $\chi_j^0$  are the mass eigenstate of the matrix (2.6) and  $h$  is a  $3 \times 2$  matrix and is given by

$$h = \begin{pmatrix} Y_1^\Sigma & Y_1^N \\ Y_2^\Sigma & Y_2^N \\ Y_3^\Sigma & Y_3^N \end{pmatrix} \cdot V(\alpha) . \quad (2.19)$$

where  $V(\alpha)$  is the  $2 \times 2$  orthogonal matrix that diagonalizes the matrix in equation (2.6). There are two contributions to the neutrino masses from the loops in figure 1, where the  $\eta_0$  and  $\eta_A$  fields are involved in the loop. With the above ingredients, from the diagram in Fig. 1 one finds that the neutrino mass matrix is given by:

$$M_{\alpha\beta}^\nu = \sum_{k=1,2} \frac{h_{\alpha\sigma} h_{\beta\sigma}}{16\pi^2} I_k (M_k, m_{\eta_0}^2, m_{\eta_A}^2) . \quad (2.20)$$

The  $I_k$  functions correspond essentially to a differences of the  $B_0$  Veltman functions [14], when evaluated at different scalar masses, note they have mass dimensions. The index  $k$  runs over the  $\chi^0$  mass eigenvalues, i.e.  $\sigma = 1, 2$ . Note that these masses are independent of the renormalization scale. In the equation below, each  $M_k$  stands for the mass values of the  $\chi^0$  fields.

$$I_k (M_k, m_{\eta_0}^2, m_{\eta_A}^2) = M_k \frac{m_{\eta_0}^2}{m_{\eta_0}^2 - M_k^2} \log \left( \frac{m_{\eta_0}^2}{M_k^2} \right) - M_k \frac{m_{\eta_A}^2}{m_{\eta_A}^2 - M_k^2} \log \left( \frac{m_{\eta_A}^2}{M_k^2} \right) \quad (2.21)$$

It is useful to rewrite the equation 2.20 in a compact way as follows

$$M^\nu = h v_h \cdot \begin{pmatrix} \frac{I_1}{16\pi^2 v_h^2} & 0 \\ 0 & \frac{I_2}{16\pi^2 v_h^2} \end{pmatrix} \cdot h^T v_h \equiv h v_h \cdot \frac{D_I}{v_h^2} \cdot h^T v_h \sim m_D \frac{1}{M_R} m_D^T \quad (2.22)$$

which is formally equivalent to the standard type-I seesaw relation with  $M_R^{-1} \rightarrow D_I/v_h^2$  [15]. This is a diagonal matrix while  $h v_h$  plays the role of the Dirac mass matrix, in our case it is a  $3 \times 2$  matrix. It is not difficult to see that we can fit the required neutrino oscillation parameters [1, 2], for example, by means of the Casas Ibarra parametrization [16].

In order to get an idea about the order of magnitude of the parameters required for producing the correct neutrino masses, one can consider a special limit in equation 2.20. For example, in cases where both  $\chi^0$  are lighter than the other fields, we have from 2.20:

$$M_{\alpha\beta}^\nu = \sum_{\sigma=1,2} \frac{h_{\alpha\sigma} h_{\beta\sigma}}{8\pi^2} \frac{\lambda_5 v_h^2}{m_0^2} M_k . \quad (2.23)$$



Parameter	Range
$M_N$ (GeV)	$1 - 10^5$
$M_\Sigma$ (GeV)	$100 - 10^5$
$m_{\eta^\pm}$ (GeV)	$100 - 10^5$
$M_\pm$ (GeV)	$100 - 10^4$
$ \lambda_i $	$10^{-4} - 1$
$ \lambda_i^{\eta,\Omega} $	$10^{-4} - 1$
$ Y_i $	$10^{-4} - 1$

**Table 2.** Scanning parameter ranges. The remaining parameters are calculated from this set.

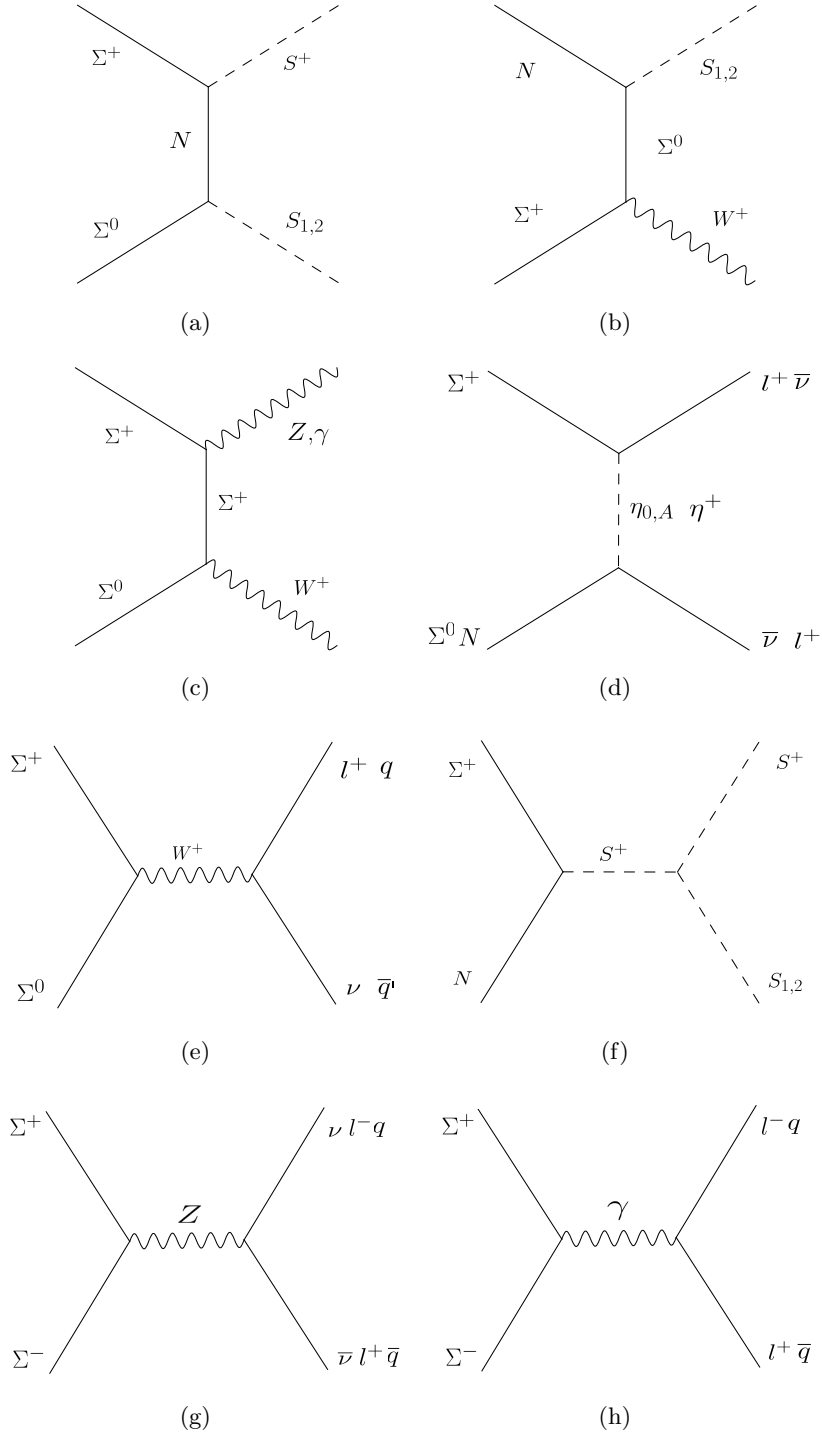
Here  $\lambda_5$  is the  $(\phi^\dagger \eta)^2$  coupling introduced in equation 2.11. The  $M_k$  are the masses of the neutral  $Z_2$  fermion fields  $\chi$ . The  $m_0$  mass term comes from writing the masses of the  $\eta_0$ , and  $\eta_A$  in the following way:  $m_{\eta_0, \eta_A}^2 = m_0 \pm \lambda_5 v_h^2$ , see appendix A.1 for more details. In particular we are interested in the magnitude of the Yukawa couplings  $h_{\alpha\beta}$  required in order to have neutrino with masses of the order of eV. For masses of  $\chi^0$  of order of 10 GeV and  $\eta_{0,A}$  of order of 1000 GeV, and  $\lambda$  couplings not too small, namely of order of  $10^{-2}$ , one finds that the values for  $h_{\alpha\beta}$  are in the order of the bottom Yukawa coupling  $\sim 10^{-2}$ . Hence it is not necessary to have a tiny Yukawa for obtaining the correct neutrino masses.

### 3 Fermion Dark Matter

As previously described the model contains two classes of potential dark matter candidates. One class are the  $Z_2$  odd scalars:  $\eta^0$  and  $\eta^A$ , when any of them is the lightest  $Z_2$  odd particle. Their phenomenology is very close to the inert doublet dark matter model [17] or discrete dark matter models [18, 19]. For this reason here we focus our analysis on the other candidates which are the fermion states  $\chi_i^0$ . In this case, the dark matter candidate is a mixed state between  $N$  and  $\Sigma^0$ . This interplay brings an enriched dark matter phenomenology with respect to models with only singlets or triplets.

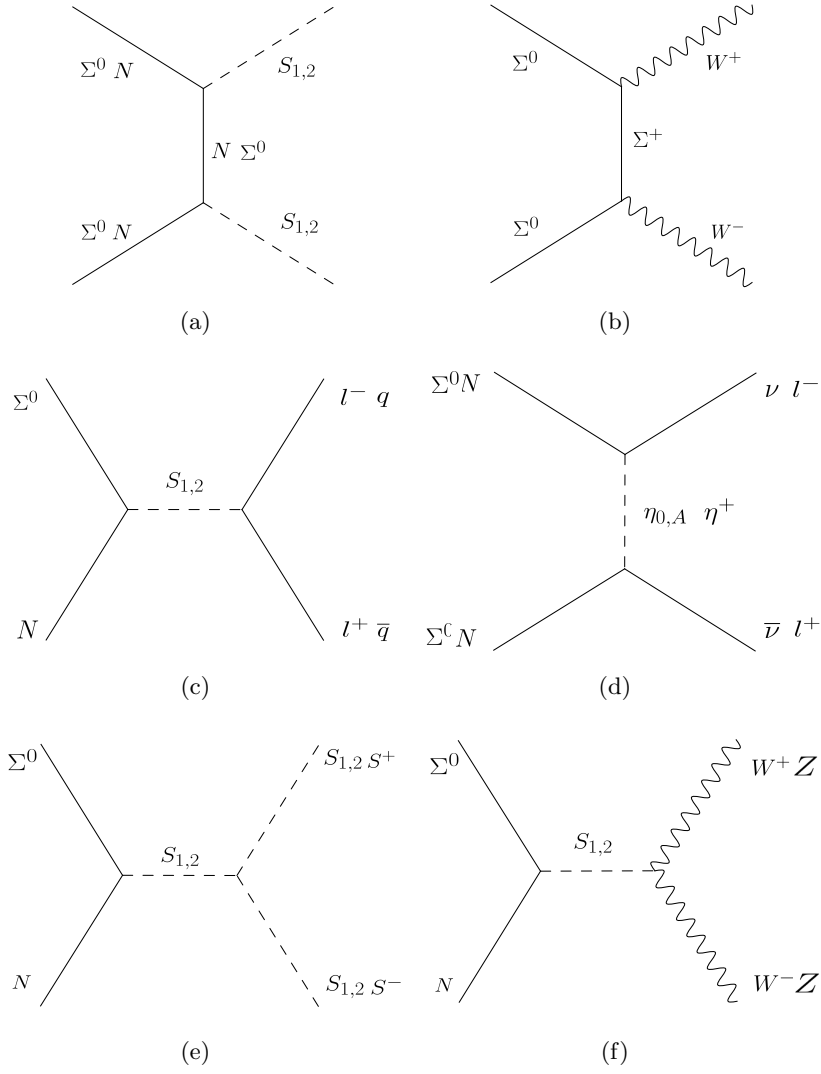
For models with only fermion triplets as dark matter, equivalent in our model to taking  $M_N \rightarrow \infty$ , the main constraints come from the observed relic abundance (equation 1.5). Coannihilations between  $\Sigma^0$  and  $\Sigma^\pm$  are efficient processes due to the mass degeneracy between them, controlling the relic abundance. These processes force the dark matter mass to be 2.7 TeV. In addition, direct detection occurs only at the one loop level [20], see Fig. 4. Most of the corresponding features have been already studied in [6, 21]. In figure 2, we show the coannihilation channels present in our model in terms of gauge eigenstates, except for the  $Z_2$  even scalars. The dark matter mass can be much smaller for singlets fulfilling the  $\Omega_{\text{DM}} h^2$  constraint. However, processes related to direct detection are absent at tree level [22] for singlets too.

The presence of the scalar triplet  $\Omega$  and its nonzero vev induces a mixing between  $\Sigma^0$  and  $N$ , implying coannihilations that can be important when the dark matter has a large component of  $\Sigma^0$ . This mixing also breaks the degeneracy between the mass eigenstate fermions  $\chi_1^0$  and  $\chi^\pm$ . However, in this case, the mass degeneracy with the charged fermion

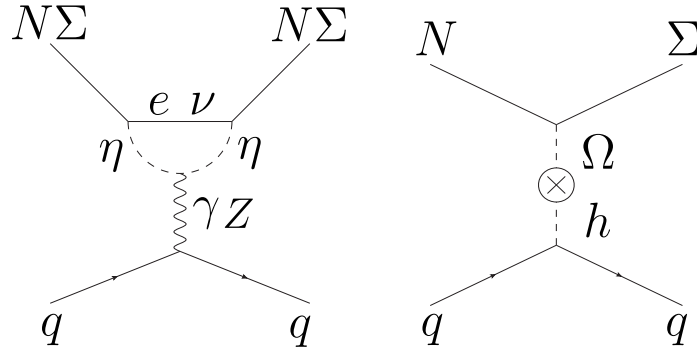


**Figure 2.**  $\Sigma^0$  and  $N$  co-annihilation channels. Figures (g) and (h) correspond to the processes involved in the  $\Sigma^\pm$  abundance.

$\chi^\pm$  is increased and forces the dark matter to be  $\mathcal{O}(\text{TeV})$ . Other coannihilation processes occur when  $M_N$  is also degenerate with  $M_\Sigma$ . For the opposite case, when  $\chi^0$  is mainly



**Figure 3.**  $\Sigma^0$  and  $N$  annihilation channels.



**Figure 4.** Direct detection in pure triplet or pure singlet models (left panel) and in our mixed triplet-singlet case (right panel).

$N$ , the model reproduces the phenomenology of the fermion singlet dark matter where the main signature is the annihilation into neutrinos and charged leptons (as in leptophilic dark matter) without any direct detection prospective [22]. The potential scenarios present in the model have the best of singlet-only or triplets-only scenarios and more. In addition, the dark matter phenomenology includes new annihilation and coannihilation channels when kinematically accessible.

The presence of the scalar triplet  $\Omega$  also induces an interaction between dark matter and quarks (direct detection) via the exchange of neutral scalar  $S_i(h^0, \Omega^0)$ , as illustrated in In Fig 3, we show the main diagrams of the model related to indirect and direct searches. The model can potentially produce the typical annihilation channels appearing in generic weakly interactive massive particle dark matter models. Indeed, our dark matter candidate mimicks the Lightest Supersymmetric Particle (neutralino) present in supergravity-like versions the Minimal Supersymmetric Standard Model with R-parity conservation. The latter would correspond here to our assumed  $Z_2$  symmetry.

In order to study the dark matter phenomenology, we have implemented the lagrangian (equation 2.5) using the standard codes `LanHEP` [23–25] and `Micromegas` [26]. We scan the parameter space of the model within the ranges indicated in Tab. 2. We also take into account the following constraints: perturbativity and a Higgs-like scalar at  $\sim 125$  GeV. Also we take into account the constraints from the relic abundance [10] as well as the lower bound on the masses of new non-colored charged particles coming from LEP [8] and LHC [9] collider searches, roughly translated to  $M_{\text{LEP}} > 100$  GeV. We calculate the thermally averaged annihilation cross section  $\langle\sigma v\rangle$ , and the spin independent cross section  $\sigma_{\text{SI}}$ .

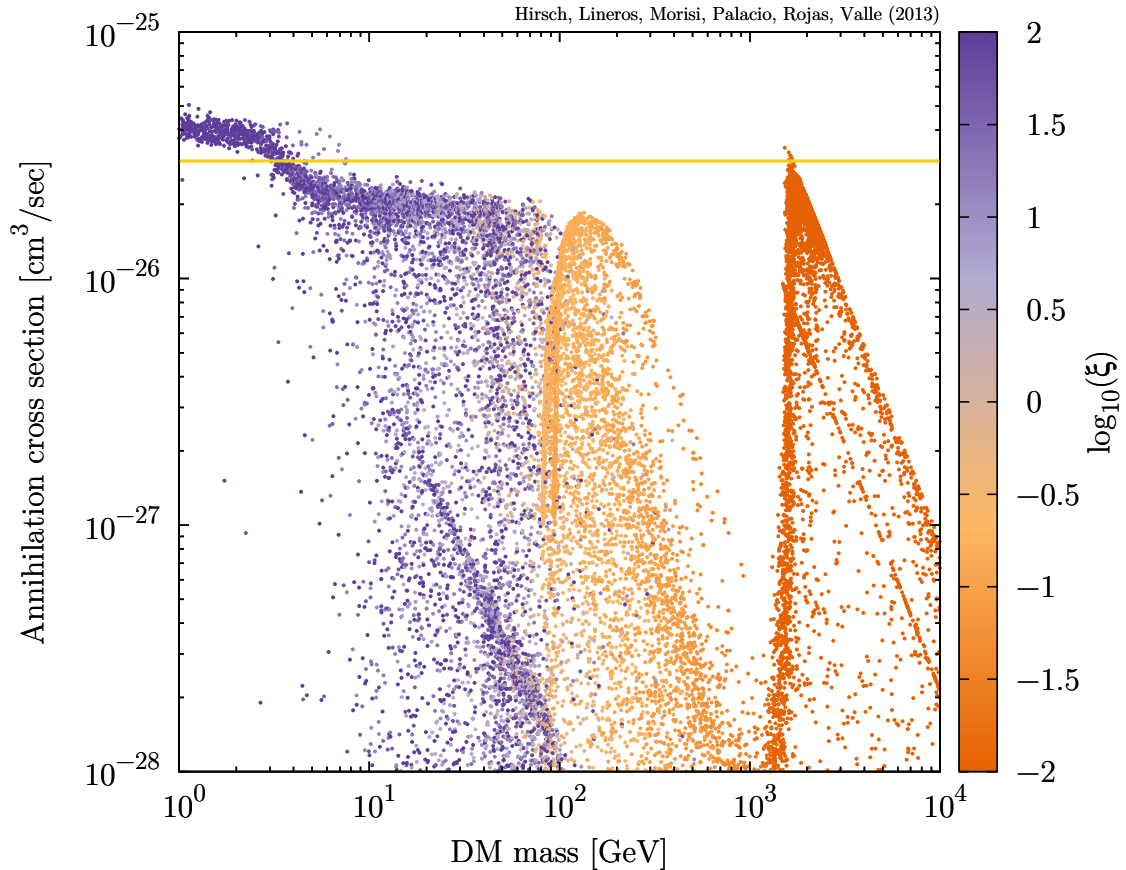
In figure 5, we present the results of the scan in terms of the annihilation cross section versus the dark matter mass. Moreover, we show in color scale the quantity:

$$\xi = \frac{M_\Sigma - m_{\text{DM}}}{m_{\text{DM}}}, \quad (3.1)$$

which estimates how degenerate is the dark matter mass with respect to  $M_\Sigma$ . Small values of  $\xi$  imply dark matter with a large component of  $\Sigma^0$  and large value implies a large component of  $N$ . This quantity has implications for coannihilation processes discussed previously. We notice that regions with low dark matter masses ( $< 20$  GeV) are less degenerate mainly because  $M_\Sigma > M_{\text{LEP}}$ . In this region the dark matter contains a large component of  $N$ . As expected, the TeV region is dominated by dark matter with large component of  $\Sigma^0$ . The mass range 100–800 GeV is particularly interesting because any of the new charged particles are accessible at LHC. Moreover, when the  $\Sigma^0/N$  mixing is non-zero and  $m_{\text{DM}} \simeq \frac{m_{S_i}}{2}$ , the annihilation channels into quarks and leptons are naturally enhanced due to the  $s$ -channel resonance in the process:

$$\chi_1^0 \chi_1^0 \rightarrow S_i \rightarrow f \bar{f} \rightarrow \langle\sigma v\rangle \propto \left( \frac{\sin(2\alpha)}{(2m_{\text{DM}})^2 - m_{S_i}^2} \right)^2. \quad (3.2)$$

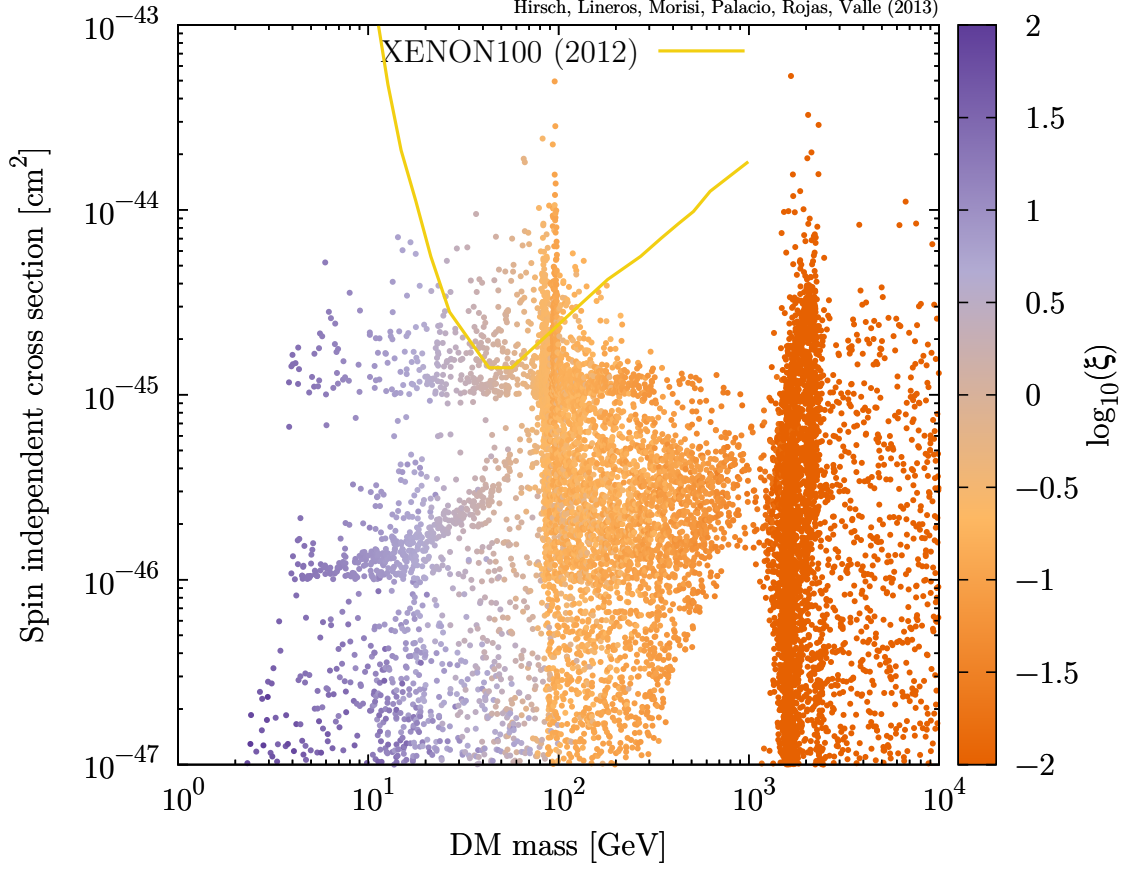
This is translated into higher expected fluxes of gamma-rays and cosmic-rays for indirect searches as well as higher spin independent cross section.



**Figure 5.** Annihilation cross section vs dark matter mass. Color scale represents  $\log_{10}(\xi)$ . Dark matter with masses larger than 1 TeV have a larger component of  $\Sigma^0$ , cases with masses lower than 20 GeV have larger component of  $N$ . The yellow line corresponds to the thermal value  $3 \times 10^{-26} \text{ cm}^3/\text{sec}$ .

Now, turning to the direct detection perspectives, the plot of the spin-independent cross section versus the dark matter mass is shown in figure 6. The scattering with quarks is described only with one diagram (the exchange of scalars  $S_i$ ), also shown in figure 4. The size of the interaction will depend directly on the mixing  $\Sigma^0/N$ . For masses larger than 100 GeV, we observe an increase of  $\sigma_{SI}$  because maximal mixing can be obtained for  $M_N \sim M_\Sigma$  and for  $Y_\Omega v_\Omega \neq 0$ . This does not occur for masses much lower to 100 GeV since the dark matter becomes mainly a pure  $N$ . Moreover, the model produces  $\sigma_{SI}$  large enough to be observed in direct detection experiments such XENON100 [27] (yellow line).

Finally, we note that the new particles introduced in our model can be kinematically accessible at the LHC. Here we briefly comment on relevant production cross sections for the LHC. Both, ATLAS [28] and CMS [9] have searched for pair production of heavy triplet fermions:  $\Sigma^0 + \Sigma^+$ , deriving lower limits on  $m_{\Sigma^+}$  of the order of  $m_{\Sigma^+} \gtrsim (180 - 210) \text{ GeV}$  [9] and  $m_{\Sigma^+} \gtrsim 245 \text{ GeV}$  [28], respectively. However, these bounds do not apply to our model, because the final state topologies used in these searches, tri-leptons in case of CMS [9] and four charged leptons in ATLAS [28], are based on the assumption that  $\Sigma^0$  decays to



**Figure 6.** Spin independent cross section vs dark matter mass. Color scale is the same as in figure 5. The yellow line is the upper bound from XENON100 experiment [27].

the final states  $\Sigma^0 \rightarrow l^\pm l^\mp + \nu/\bar{\nu}$ . As a result of the  $Z_2$  symmetry present in our model, however, the lightest fermion or scalar is stable and all heavier  $Z_2$ -odd states will decay to this lightest state. Thus, the intermediate states  $\Sigma^0 + \Sigma^+$  and  $\Sigma^- + \Sigma^+$ , which have the largest production cross sections of all new particles in our model, will not give rise to three and four charged lepton signals.

Instead, the phenomenology of  $\Sigma^0$  and  $\Sigma^+$  depends on the unknown mass ordering of fermions and scalars. Since we have assumed in this paper that the lighter of the fermions is the dark matter, we will discuss only this case here. Then, the phenomenology depends on whether the lightest of the neutral fermions,  $\chi_1^0$ , is mostly singlet or mostly triplet. Consider first the case  $\chi_1^0 \simeq \Sigma^0$ . Then, from the pair  $\chi_1^0 + \Sigma^+$ , only  $\Sigma^+$  decays via  $\Sigma^+ \rightarrow \chi_1^0 + W^+$ , where the  $W^+$  can be on-shell or off-shell. Thus, the final state consists mostly one charged lepton plus missing energy. The other possibility is pair production of  $\Sigma^+ + \Sigma^-$  via photon exchange, which leads to  $l^+ + l^-$  plus missing energy. In both cases, standard model backgrounds will be large and the LHC data probably does not give any competitive limits yet. We expect that LHC data at 14 TeV with increased statistics may constrain part of the parameter space. A quantitative study would require a MonteCarlo analysis which is beyond the scope of this work.

Conversely, for the case  $\chi_2^0 \simeq \Sigma^0$ , the  $\chi_2^0$  will decay to  $\chi_1^0$  plus either one on-shell or off-shell Higgs state, depending on kinematics. In this case the final state will be one charged lepton plus up to four b-jets plus missing momentum. This topology is not covered by any searches at the LHC so far, as far as we are aware.

Also, the new neutral and charged scalars can be searched for at the LHC. All possible signals have, however, rather small production cross sections. Neither  $\eta$  nor  $\Omega$  have couplings to quarks and only  $\Omega$  (both charged and neutral) can be produced at the LHC due to its mixing with the Standard Model Higgs field  $\phi$ . Final states will be very much SM-Higgs like, but the event numbers will depend quadratically on this mixing, which supposedly is a small number, since the observed state with a mass of roughly (125 – 126) GeV behaves rather closely like A Standard Model Higgs. Searches for a heavier state with Standard Model like Higgs properties [29] exclude scalars with standard coupling strength now up to roughly 700 GeV. However, upper limits on  $\sin^2(\theta)$  in the mass range (130 – 700) GeV are currently only of the order (0.2 – 1.0). The next run at the LHC, with its projected luminosity of order  $\mathcal{L} \simeq (100 - 300) \text{ fb}^{-1}$ , should allow to probe much smaller mixing angles.

## 4 Conclusions

We have presented a next-to minimal extension of the Standard Model including new  $Z_2$ -odd majorana fermions, one singlet  $N$  and one triplet  $\Sigma$  under weak  $SU(2)$ , as well as a  $Z_2$ -odd scalar doublet  $\eta$ . We also include a  $Z_2$ -even triplet scalar  $\Omega$  in order induce the mixing in the fermionic sector  $N$ – $\Sigma$ . The solar and atmospheric neutrino mass scales are then generated at one-loop level, with the lightest neutrino remaining massless. This way our model combines the ingredients present in Refs. [5, 6] with a richer phenomenology.

The unbroken  $Z_2$  symmetry implies that the lightest  $Z_2$ -odd particle is stable and may play the role of dark matter. We analyze the viability of the model using state-of-art codes for dark matter phenomenology. We focus our attention to the fermionic dark matter case. The mixing between  $N$  and the neutral component of  $\Sigma$  relaxes the effects of coannihilations between the dark matter candidate and the charged component of  $\Sigma$ . In the pure triplet case, the dark matter mass is forced to be 2.7 TeV in order to reproduce the observed dark matter abundance value. However, in the presence of mixing the effect of coannihilations is weaker, allowing for a reduced dark matter mass down to the GeV range. Thanks to that, the charged  $\Sigma$  can be much lighter than in the pure triplet case, opening the possibility of new signatures at colliders such as the LHC. In addition, the dark matter candidate can interact with quarks at tree level and then produce direct detection signal that may be observed or constrained in current direct searches experiments such XENON100.

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## A Appendix

### A.1 Approximations for Neutrino Masses.

Starting from the equation 2.20, one can perform some approximations to examine neutrino masses for cases of interest, for example, cases with one of the  $\chi_1^0$  masses being the lightest between  $\chi_2^0$ ,  $\eta_{0,A}$ ,  $\Sigma_{\pm}$  and  $\Omega_{0,\pm}$ .

One wants to establish the relation between neutrino masses and the other parameters in the lagrangian in a suitable form. In principle, neutrino masses depend on the masses of neutral  $\eta$  fields and the masses of the  $\chi^0$ , but the dependence of the parameters of the scalar sector is more complicated, given the structure of the masses of the  $\eta$  fields (see equations 2.16 and 2.17). One can take these equations and write them in the following way:

$$m_{\eta_0}^2 = m_0^2 + \lambda_5 v_h^2, \quad (\text{A.1})$$

$$m_{\eta_A}^2 = m_0^2 - \lambda_5 v_h^2. \quad (\text{A.2})$$

Where  $m_0^2$  is a complicated function of the parameters of the scalar potential. One can write the equation 2.21 as follows:

$$\begin{aligned} I_k = & -M_k \left( \frac{m_0^2 + \lambda_5 v_h^2}{M_k^2 - m_0^2 - \lambda_5 v_h^2} \right) \log \left( \frac{m_0^2 + \lambda_5 v_h^2}{M_k^2} \right) \\ & + M_k \left( \frac{m_0^2 - \lambda_5 v_h^2}{M_k^2 - m_0^2 + \lambda_5 v_h^2} \right) \log \left( \frac{m_0^2 - \lambda_5 v_h^2}{M_k^2} \right). \end{aligned} \quad (\text{A.3})$$

One can identify two interesting limit cases. When  $\lambda_5 v_h^2 \ll M_k^2 \approx m_0^2$  then the  $I_k$  function can be written as:

$$I_k = \frac{2\lambda_5 v_h^2}{M_k}. \quad (\text{A.4})$$

Therefore, the neutrino mass matrix in this approximation is given by:

$$M_{\alpha\beta}^\nu = \sum_{\sigma=1,2} \frac{h_{\alpha\sigma} h_{\beta\sigma}}{8\pi^2} \frac{\lambda_5 v_h^2}{M_\sigma}. \quad (\text{A.5})$$

The other case is given by  $\lambda_5 v_h^2, M_k^2 \ll m_0^2$ , the procedure is not difficult, the result is:

$$I_k = \frac{2\lambda_5 v_h^2}{m_0^2} M_k. \quad (\text{A.6})$$

In this case, the neutrino mass matrix is given by:

$$M_{\alpha\beta}^\nu = \sum_{\sigma=1,2} \frac{h_{\alpha\sigma} h_{\beta\sigma}}{8\pi^2} \frac{\lambda_5 v_h^2}{m_0^2} M_k. \quad (\text{A.7})$$



## A.2 Minimization conditions

The tadpole equations were computed in order to find the minimum of the scalar potential, thus, the linear terms of the scalar potential at tree level can be written as:

$$V_{(1)} = t_h h_0 + t_\eta \eta_0 + t_\Omega \Omega_0 \quad (\text{A.8})$$

Where the tadpoles are:

$$t_h = v_h \left( -m_1^2 + \frac{1}{2} \lambda_1 v_h^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_\eta^2 \right) \quad (\text{A.9})$$

$$t_\eta = v_\eta \left( m_2^2 + \frac{1}{2} \lambda_2 v_\eta^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_h^2 \right) \quad (\text{A.10})$$

$$t_\Omega = -M_\Omega^2 v_\Omega - \mu_1 v_h^2 + (2\lambda_1^\Omega + \lambda_4^\Omega) v_h^2 v_\Omega + 8 (2\lambda_2^\Omega + \lambda_3^\Omega) v_h^2 v_\Omega^3 + \mu_2 v_\eta^2 + (2\lambda_1^\eta + \lambda_4^\eta) v_\Omega^2 v_\eta^2 \quad (\text{A.11})$$

In order to have an  $Z_2$  invariant vacuum, the vev  $v_\eta$  has to vanish, which is extracted from the equation A.10. For the vev  $v_h$ , one can choose the value to be nonzero solving the equation in the parenthesis, in equal manner, one obtain the vev  $v_\Omega$ , in terms of the other parameters of the potential.

The numerical values of the vevs  $v_h$  and  $v_\Omega$  are restricted to reproduce the measured values of gauge boson masses, this allows to have the value for  $v_h \sim 246$  GeV, and  $v_\Omega \leq 7$  GeV, as one can see in the section 2.3.

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